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IMAGE DENOISING USING HARD THRESHOLD TECHNIQUES ON WAVELET TRANSFORM AND SHEARLET TRANSFORM

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Abstract- In this data age century with increment in the modern technology there is a development in the theory of multidimensional data to provide the higher directional sensitivity in imaging. A numeric image is a portrayal of a real image which is taken as a set of numbers that can be gathered and picked up by a digital computer. In order to decode the image into numbers it is divided into small segments called pixels (picture elements). Whenever there is a transmission of images or due to some environment factor there is an addition of noise to the images takes place that ultimately results in the reduction of originality of the image. It is very important to remove the noise from the images so that it is safeguard. Shearlets are a multiscale foundation which authorize efficient encoding of anisotropic feature in multivariate problem classes. In this paper, we have set forth the noise removal transform by hard thresholding for denoising. We can denoise the noisy image by wiping out the fine details, to enhance the quality of the images.

Keywords: Wavelets, Wavelet Transform, Shearlet, Shearlet Transform, Image De- noising.

1. INTRODUCTION

Digital images are the aggregation of pictures elements called pixel. Each pixel contains location and intensity values. These Digital images get corrupted or noisy by the imaging device or due to external environment in transmission or digitization procedures, then the resultant image is called noise image and to maintain the originality of the image it is very important to eliminate the noise from the image. The process by which elimination of unwanted noise from the images can take place is called Image Denoising [1]. The de- noised image contain less noise than the observation while still keeping sharp transitions (i.e edges)[2]. Image processing systems are so successful and popular too because of the easy inherence of personnel computers, large size memory devices, graphics software etc. The main problem is that the image get noisy due to the acquisition and transmission. In history, wavelet was firstly discovered by Grossman and Morlet, they created the geometrical decorum of the continuous wavelet transform based on in-variance under the affine group known as translation and dilation which authorize the decomposition of a signals into contributions of both space and scale [3]. The complex valued generalization of shear- let transform was originally introduced by Kutyniok, Lbate, Lim ans Weiss in 2005 and applies anisotropically escalade. Shearlet works efficiently in higher dimensions. Shearlet transforms are the upgraded class of wavelets and provide improved results as compared to wavelets. While denoising the image, the edges is not preserved in case of wavelet transform but shearlet are efficient in preserving edges of an image in image denoising. So, shearlet comes out as a efficient transform for edges analysis and detection. For the denoising and enhancement of the image we use both wavelet and shearlet transform in this paper. This paper has a comparative study between wavelet and shearlet transform. This comparative study between these two transforms will help to build a deeper understanding of the con- demnation techniques. We can remove the unwanted signals in the videos which are the cause of the denoised videos, just like the noise in the images. This can be done by both wavelet and shearlet transforms. Basically, we have denoised the image using wavelet and shearlet transform in pyshear lab(python) and calculated the PSNR and MSE values for the various denoised images and hence came to the conclusion.

2. WAVELET THEORY

Wavelets were first initiated by Morlet and Grossman[4] in the early 1980s. It is obtained from “ondelette” the French word which means “small wave” developed from the review of Time-frequency signal scanning and wave propagation, sampling theory. Morlet first initiated the intention of wavelets as a group of functions established by using translation and dilation of a single function, called Mother Wavelets, for the study of non-stationary signals. Wavelets summarized as a mathematical tool which can be taken to retrieve in- formation from various types of data, together with audio signals and images. Wavelets as a subject is attaining a lot of attention from various mathematical scientists in different fields. It is producing a customary route between mathematicians, physicists and electrical engineers with modern application as varied as image processing, pattern recognition, data compression, wave propagation, medical image fusion and other computer graphics. Wavelets is defined as the mathematical function that chop up the data into the different frequency units and then study each unit according to their matched lamina wavelets that highlights the image’s scale of feature and position which can be applied to 1D signals.

From the chronicled frame of reference, wavelet transform anatomizing is a new born process. Wavelet Analysis shows may different origins[5]. Most of the work was done in the 1930s and at that time, the individual efforts did not arise to be parts of a coherent theory. However, its mathematical development accelerates to the research to Joseph Fourier in 1807. Fourier allocate the base by his theories based on frequency analysis which certified to be extremely powerful and pertinent.

He postulated that any 2π-periodic function g(x) is the sum

$$\mathbf{P_0 + \sum_{K=1}^{\infty}(P_k cos(kx) + q_k sin(kx))} \quad (2.1)$$

of its fourier series. The coefficients po, pk and qk are calculated by

$$\mathbf{P_0 = \frac{1}{2\pi}\int_0^{2\pi} g(x)dx} \quad (2.2)$$

$$\mathbf{P_k = \frac{1}{\pi}\int_0^{2\pi} g(x)\cos(kx)\,dx} \quad (2.3)$$

$$\mathbf{q_k = \frac{1}{\pi}\int_0^{2\pi} 2\pi g(x)\sin(kx)\,dx} \quad (2.4)$$

Fourier's assertion played an important role in the advancement of the ideas mathematicians had about the functions. He gave new dimension to a new functional universe. Wavelets has come to spotlight in 1909 in a thesis which was given by Alfred Haar. After a year, in 1910, Alfred Haar gives the advancement of a set of a rectangular basis functions. Then after two decades, In 1930, paul levy audit The Brownian Motion[5]. Littlewood and paley explored the area where function's contribution energy was studied

$$\text{Energy} = \mathbf{\frac{1}{2}\int_0^{2\pi} |g(x)|^2\,dx} \quad (2.5)$$

In 1946, Dennis Gabor look over short time fourier transform. Then in 1975, George Zweig gave the first continuous wavelet transform. In 1985, Meyer builded the orthogonal wavelet basis function that having very advantageous time and frequency localization. In 1986[6] Stephane Mallat work in developing the proposition of multiresolution analysis for DWT. The existing conceptual notion was first given by Jean Morlet and the group at the Marseille. Theoretical physics center working under Alex Grossman in France. The strategy of wavelet analysis was created mainly by Y. Meyer and his collaborator, who have established the process broadcasting. The chief algorithm date back to the concept of stephane Mallat in 1986[7]. And in 1992, Albert cohes and Daubechies constructed the compactly supported biorthogonal wavelets. Since then, research on wavelets has become worldwide. This research is incredibly active in the united states, where it is van-guarded by the work of scientists such as Ingrid Daubechies, Ronald Cofiman and victor Wickhauser.

3. WAVELET TRANSFORM

The wavelet transform was first introduced in the context of a mathematical Transform by Grossman and and Morlet in 1984. It is defined as the mathematical function which is utilize to separate a designated function or continuous-time signal into various scale component. It is a tool which allow us to review each component with a resolution that can match to the lamina. One can set a frequency range to each lamina component. It is the portrayl of a function by wavelets. Wavelets are the scaled and translated copies (known as "Daughter Wavelets") of a finite-length or fast decaying oscillating waveform (known as the "Mother Wavelet")

$$\Phi_{\mathbf{p,q}}(\mathbf{t}) = \mathbf{\frac{1}{\sqrt{|p|}}}\,\Phi\left(\mathbf{\frac{t-q}{p}}\right), \mathbf{p,q} \in \mathbf{R}, \mathbf{p} \neq \mathbf{0} \quad (3.1)$$

where, φ is the wavelet function, p is the scaling parameter which measure the degree of compression or q is the translation parameter which determined the time location of the wavelet [8,23].

3.1 Discrete Wavelet Transform

There is no need to calculate the wavelet coefficients at each scale. Instead that is enough to choose scales which are based on power of two and we can get the similar results.

$$\boldsymbol{\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi(2^j x - k)} \quad (3.2)$$

If the function is expanding like samples of a continuous function g(x), the output coefficients are called DWT of g(x). In this case, One dimensional series expansion of wavelet transform in is presented as follows.

$$\mathbf{W_\Phi(j_0, k) = \frac{1}{\sqrt{R}}\sum_x g(x)\,\Phi_{j_0,k}(x)} \quad (3.3)$$

$$\mathbf{W_\Phi(j, k) = \frac{1}{\sqrt{R}}\sum_x g(x)\,\boldsymbol{\psi}_{j,k}(x)} \quad (3.4)$$

for $j \geq j0$ and

$$\mathbf{G(x) = \frac{1}{\sqrt{R}}\sum_x W_\Phi(j_0,k)\,\Phi_{j_0,k}(x) + \frac{1}{\sqrt{R}}\sum_{j=j_0}^{\infty}\sum_k W_\psi(j,k)\boldsymbol{\psi}_{j,k}(x)} \quad (3.5)$$

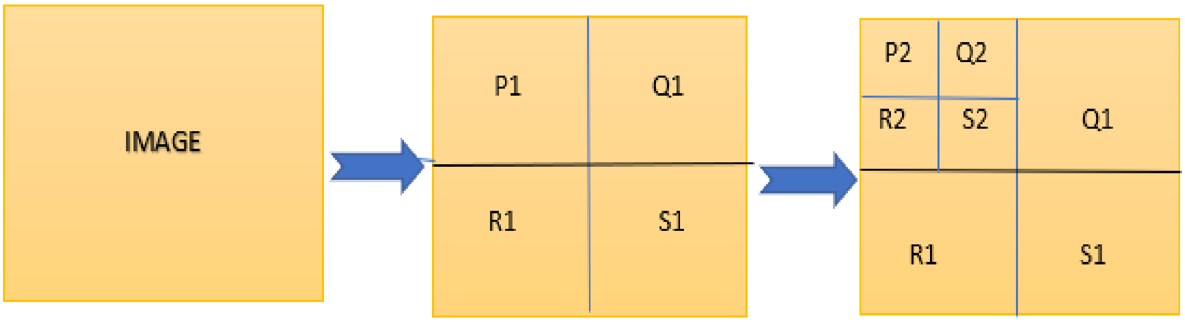
where g(x), $\Phi_{j_0,k}(x)$ and $\boldsymbol{\psi}_{j,k}(x)$ are the functions of the discrete variable x = 0, 1, 2......R-1. The coefficients given in equation (8) and (9) are known as the approximation and detail coefficients respectively [9]. The Discrete Wavelet Transform in two dimension of function g(x, y) of size is as follows.

$$\mathbf{W_\Phi(j_0, r, s) = \frac{1}{\sqrt{RS}}\sum_{x=0}^{R-1}\sum_{y=0}^{S-1} f(x,y)\,\Phi_{j_0,r,s}(x,y)} \quad (3.6)$$

$$\mathbf{W_\psi^i(j, r, s) = \frac{1}{\sqrt{RS}}\sum_{x=0}^{R-1}\sum_{y=0}^{S-1} f(x,y)\boldsymbol{\psi}_{j,r,s}^i(x,y)} \quad (3.7)$$

$\mathbf{i = \{H, V, D\}}$, In case of one dimensional DWT $\mathbf{j_0}$ is an arbitrary starting lamina and the Wφ($\mathbf{j_0}$, r, s) coefficients define an approximation of g(x, y) at lamina $\mathbf{j_0}$, $\mathbf{W_\psi^i(j,r,s)}$ coefficient add vertical, horizontal and diagonal details for laminas j≥ $\mathbf{j_0}$ and i is a superscript that assumes the values H,V and D [9].

The DWT (Discrete wavelet transform) is similar to hierarchic sub-band system. firstly the image taken is changed into four pieces which are represented as P 1, Q1, R1 and S1 as shown in the fig. The P 1 sub-band is known as the approximation and it can again break into four sub-bands. The bands which are left are known as detailed



components. To get further level of decomposed sub-bands. Sub-bands is again broken. A lot of wavelets has the necessity to depict the edge number which is based on the edge's length. It does not consider the smoothness in any way. Therefore, chances are there that m-term approximation can occur[9,10,24,25].

Fig. 3.1 DWT based wavelet decomposition to various level

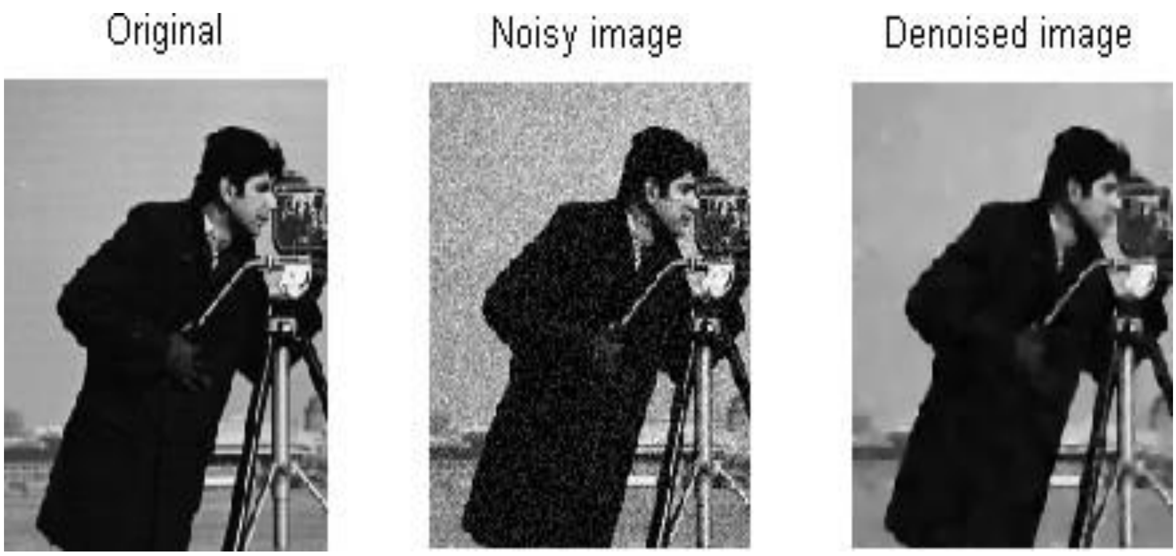


Fig. 3.2 wavelet DWT

3.2 Continuous Wavelet Transform

The continuous wavelet Transform for 1D signals g(t) is given as follows.

$$\mathbf{W_{\psi}(p,q) = \frac{1}{\sqrt{|p|}} \int_{-\infty}^{\infty} g(t)\psi * \left(\frac{t-q}{p}\right) dt} \tag{3.8}$$

This transform consisting of a function of two variables p and q where, q is the scaling framework and q is the shifting framework and ψ(t) is the mother wavelet or simply the basic function. The two frameworks p and q gives the frequency data and time data in the wavelet transform. In this type of wavelet transform the continuous function of one variable transforms into a continuous function of two variables that is translation and scale[10]. The wavelet coefficients measures the interdependence of wavelet with signal. For compact representation, select a wavelet which matches with the shape of the image components. For example-Haar wavelet for black and white drawings.

The inverse continuous wavelet Transform (ICWT) of 1D signals is given as follows.

$$\mathbf{g(t) = \frac{1}{C_{\psi}} \int_{0}^{\infty} \int_{-\infty}^{\infty} W_{\psi}(p,q)\psi_{p,q}(t) \frac{DqDp}{p^2}} \tag{3.9}$$

Where

The 2D continuous wavelet Transform of the signal g(x, y) is given as follows.

$$\mathbf{W_{\psi}(p,q) = \frac{1}{\sqrt{|p|}} \int_{-\infty}^{\infty} g(x,y)\psi * \left(\frac{x-m}{p}, \frac{y-n}{q}\right) dxdy} \tag{3.10}$$

where m, n are the shifting framework or translation framework and p is the scaling frame- work[10].

4. SHEARLET THEORY

It is clear to us that wavelet transform is not efficient in dimensions greater than one. Wavelet transform are based on the isotropic escalate that is, when dilation of a generating function occur then both dimensions are being escalated equally. On the other hand, Shearlet are based on the anisotropic escalating which dilates one direction more than the other. In the past, several new representation systems were proposed, includ ing the directional wavelets, the complex wavelets, the ridgelets, the contourlets and the curvelets. A very recent approach are shearlets, which is equipped with rich Mathematical structure similar to wavelets. The shearlet Transform method gives very good image restoration than the wavelet transform. Wavelet transform gives the output image with very much noise. But shearlet Transform gives very good result with full elimination of noises with respect to types of noises. There are some similarity between the shearlet and curvelets like both present multiscale and multidimensional analysis. Still, there are some dissimilarity exists between the two of them like shearlet transform are created by imposing a group of operators to a single function, while curvelet is not in that form. Shearlets are based on the fixed translation lattice while curvelets does not based on the fixed translation lattice. Shearlets are linked to a multiresolution scanning on the other hand, curvelets are not based on that.

5. SHEARLET

Originally, Shearlet come to this data age century in 2006 for analyzing and approximating the functions $f \in \mathbf{L^2(R^2)}$. Shearlets are the natural augmentation of wavelets. Shearlets provides the highly efficient portrayl of images with preserved edges which is difficult to obtain in case of wavelets. Shearlets are a multi-grid foundation which are mainly based on the anisotropic properties in multivariate problem classes[11].

5.1 Continuous Shearlet System and Transform

The creation of continuous shearlet transform is installed on the basis of parabolic scaling matrices.

$$\mathbf{A_a = \begin{pmatrix} a & 0 \\ 0 & a^{\frac{1}{2}} \end{pmatrix}}$$

is an anisotropic dilation matrix and it is used to control the scale of shearlet by providing the different inflation factor on two coordinate axis.

$$\mathbf{S_s = \begin{pmatrix} 1 & S \\ 0 & 1 \end{pmatrix}}$$

S is a shear matrix as a means to change the orientation and also, the shear matrix is associated with area preserving geometric transformation, such as rotation and shear for φ ∈ L2(R2). The continuous shearlet system generated by ψ is then defined as

$$\mathbf{SH_{cont}}(\Phi) = \{\Phi_{\mathbf{a,st}} = \mathbf{a^3\psi\,(S_sA_aT_t)}\} \tag{5.1}$$

The map g is defined by $\mathbf{Sh_\Phi g(a,s,t)} = <\mathbf{g}, \Phi_{\mathbf{a,s,t}}>$, $\mathbf{g \epsilon L^2(R^2)}$. The continuous shearlet transform of $g \in \mathbf{L^2(R^2)}$ is defined by

$$\mathbf{f} \mapsto \mathbf{SH_\Phi g(a,s,t)} = <\mathbf{g}, \boldsymbol{\sigma}\mathbf{(a,s,t)}\Phi> = \int \mathbf{g(x)}\boldsymbol{\sigma}\mathbf{(a,s,t)}\Phi\mathbf{(x)dx} \tag{5.2}$$

5.2 Discrete Shearlet System andTransform

Discrete shearlet system are generally formulated by sampling continuous shearlet system on a discrete subset of the shearlet group S. An irregular discrete shearlet system associated with φ and Λ represented by SH(φ, Λ) is defined by

$$\mathbf{SH}(\Phi,\boldsymbol{\lambda}) = \left\{\Phi_{\mathbf{a,s,t}} = \mathbf{a}^{-\frac{3}{4}}\Phi\left(\mathbf{A_a^{-1}S_s^{-1}}(\cdot\ -\mathbf{t})\right) : \mathbf{(a,s,t)} \in \mathbf{A}\right\} \tag{5.3}$$

Where as a regular discrete shearlet system associated with φ, is denoted by SH(φ) is defined by

$$\mathrm{SH}(\Phi) = \{\ \Phi_{\mathbf{j,k,m}} = \mathbf{2^{\frac{3}{4}j}}\left(\mathbf{S_kA_{2^j}}\ \cdot -\mathbf{m}\right) : \mathbf{j,k} \in \mathbf{Z}, \mathbf{m} \in \mathbf{Z^2}\} \tag{5.4}$$

5.3 Image Denoising

whenever there is a transmission of images takes place, then the originality of the image start degrading. In simple words, image is getting noised or noisy. Due to addition of noise, various image processing venture like image analysis and tracking, video processing are negatively posed. Hence to denoise the image is the only task left in modern image processing system. Image denoising is a process is a process which is used to detach the noise from the noisy image so that the originality of the image is maintained even after the transmissions. From the mathematical point of view image denoising is an inverse problem having the solution which is not specific. It is a classical problem that has been study for a long time. By seeing the past it is concluded that various inventive execution have been made in the section of image denoising[12,22,26,27].

Mathematically, The issue of image denoising is depicted as follows:

$$q = r + s \tag{5.5}$$

where q is the observed corrupted image, x is the unknown genuine image and s is the additive white guassian noise (AWGN)[17]. The challenges which occur while denoising the image are like textures should be safeguard, edges should be shielded without blurring, flat areas should be smooth and new product should not be formed[18,19].

The importance of insufficiency for data restoration is very well understood and has been highlighted in papers such as [20,21]. Indeed, consider the classical complication of retrieving a function $\mathbf{f} \in \mathbf{L^2(R^2)}$. from the noisy data q i.e of recovery r from observation q = r + s where s is the Guassian white noise with standard deviation σ. This is described for one dimension.

6. TYPES OF NOISES

Noise is an intolerable signal that modifies the property and efficiency of the signal. Normally, noise categories contain noise like gaussian noise, salt and pepper noise and speckle noise distribution.

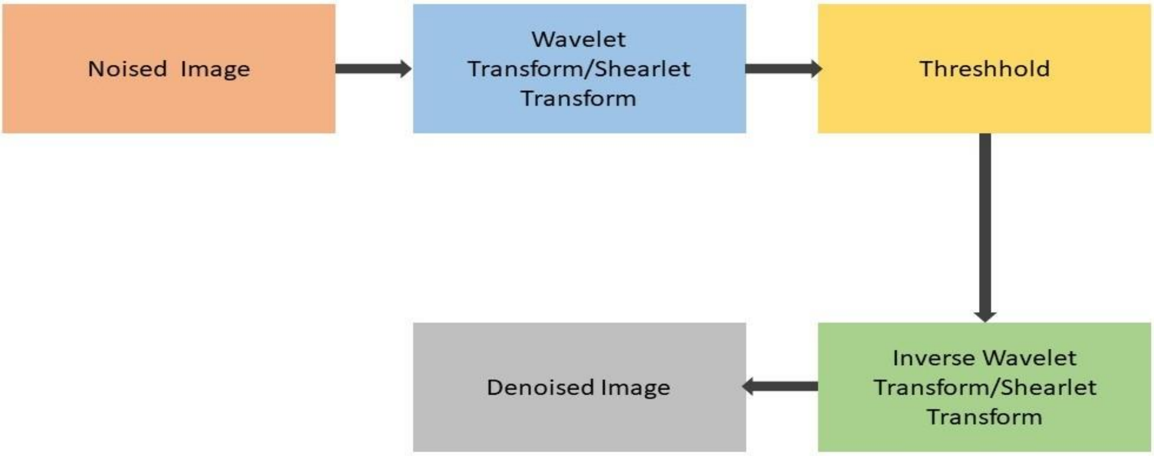


Fig. 6.1 Image denoising using wavelet/shearlet Transform

6.1 Guassian Noise

It is also known as electronic noise generally arises in elec tronic components. It is the computative noise to that of the natural distribution. The noise is free from each pixel as well as signal intensity and is protective in nature[10,13]. The Probability Density Function(PDF) 'G' of a Guassian random variable 'v' is given as follows.

$$G(v) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\mu)^2}{2\sigma^2}}$$

where,
u=Grey scale µ=mean or average value σ=Standard deviation

Fig. 6.2 Gaussian Noise

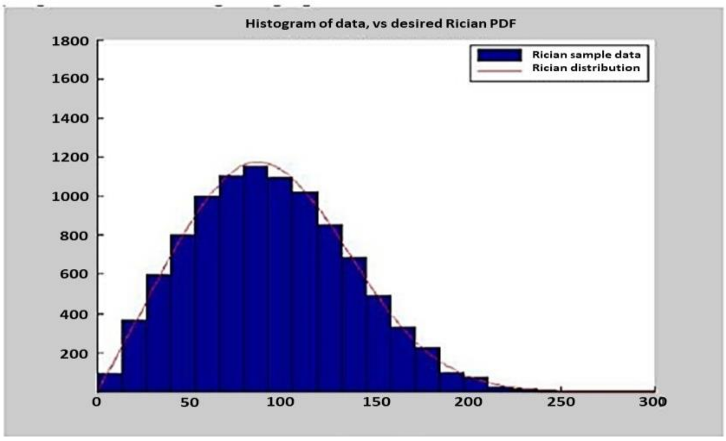


Fig. 6.3 Gaussian Graph

6.2 Salt and Pepper Noise

It is also known as Impulsive noise[10,14]. It is produced during sharp and sudden disturbances in the image and portrayl as white and black pixels. This type of noise is mainly removed by Median Filter.

The (PDF) 'S' of a Guassian random variable 'v' is given by

$$S(v) = \begin{cases} S_p & \mu = 0(\text{pepper}) \\ S_s & \mu = 2^n - 1(\text{salt}) \\ 1 - (S_p + S_s) & \mu = k(0 < k < 2^n - 1) \end{cases}$$

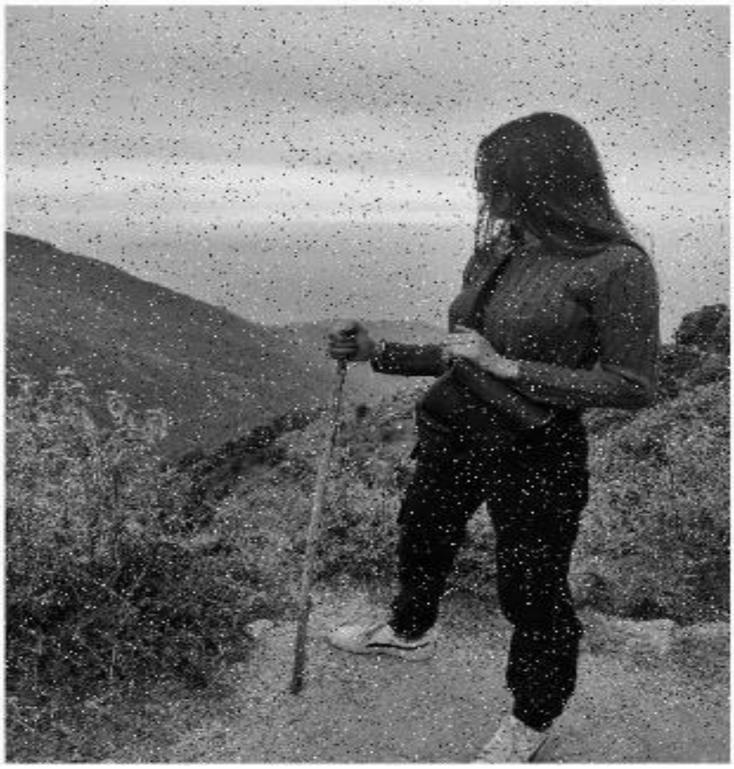


Fig. 6.4 Salt and Pepper Noise

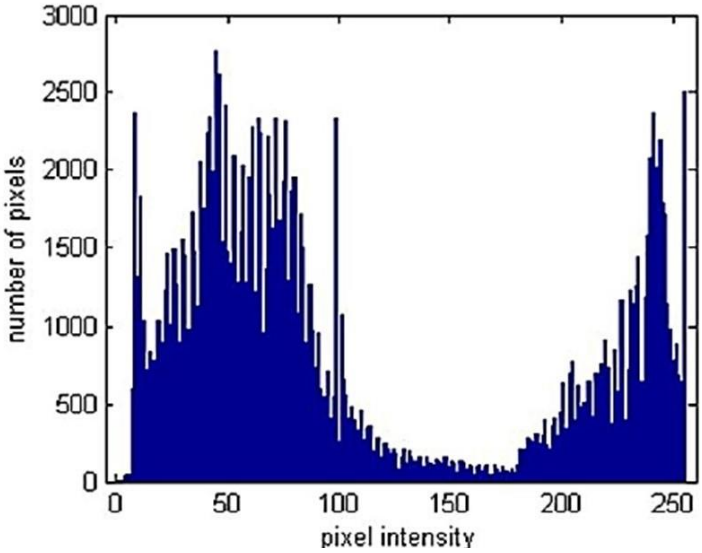


Fig. 6.5 Salt and Pepper Graph

6.3 Speckle Noise

The Quality of the medical images, optical coherence tomography and medical images are corrupted by noise called speckle noise[10,15]. The root cause of production of this type of noise is environmental factors that come into play dur- ing imaging sensor in the process of image transmission. Speckle noise follows gamma distribution which is given as follows.

$$F(v) = \frac{v^{\alpha-1)}}{(\alpha-1)!a^{\alpha}} e^{\frac{-v}{a}}$$

where,
v=Grey scale
aα=variance

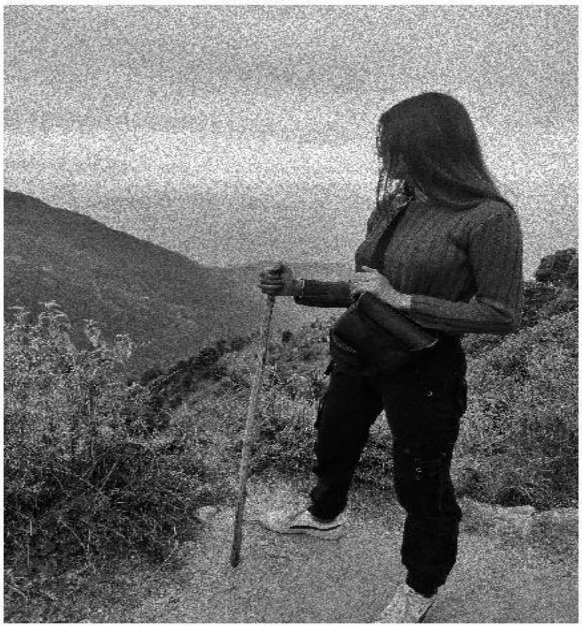


Fig. 6.6 Speckle Noise

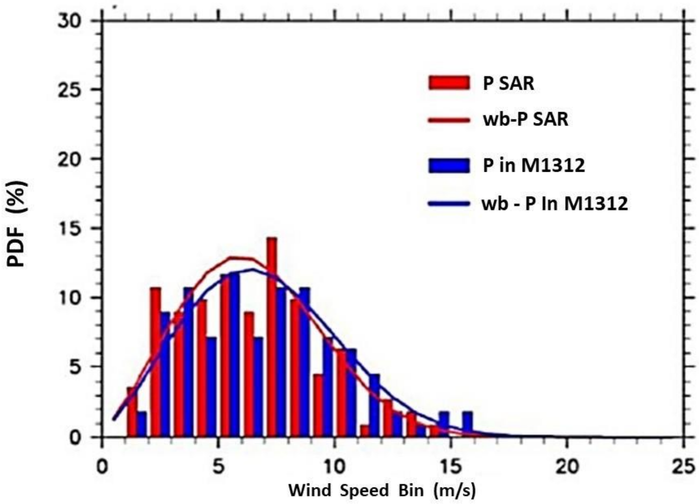


Fig. 6.7 Speckle Graph

7. PERFORMANCE PARAMETERS

Every proposed algorithm is verified using subjective analysis approach. These are performance evaluator such as PSNR, and MSE which are considered to finally quantify the result.

7.1 Peak Signal To Noise Ratio (PSNR)

PSNR[15] is defined as the ratio between the largest possible power of a signal and the power of bribed noise. PSNR measures the peak signal to noise ratio between two images which is taken as the quality measurement between two images (i.e Genuine image and Squeezed image). Higher the value of PSNR better the quality of squeezed image. It is usually denoted in term of logarithmic decibel scale.
PSNR is calculated as

$$\mathbf{PSNR_{db}} = \mathbf{10log_{10}}\left(\frac{\mathbf{MAX^2}}{\mathbf{MSE}}\right) = 20\mathbf{log_{10}}(\mathbf{MAX}) - \mathbf{10log_{10}}(\mathbf{MSE})$$

where, MAX is abbreviated as maximum possible pixel value of the image and MSE is the mean square error[15].

7.2 Mean Square Error (MSE)

The MSE [15] is defined as the cummulative inaccuracy between the squeezed image and the genuine image. Lower the MSE, better the quality of the squeezed image. It is calculated as

$$\mathrm{MSE} = \frac{\mathbf{1}}{\mathbf{ab}}\sum_{\mathbf{p=0}}^{\mathbf{a-1}}\sum_{\mathbf{q=0}}^{\mathbf{b-1}}[\mathbf{I(p,q)} - \mathbf{K(p,q)}]^{\mathbf{2}}$$

where, a, b is the dimension of the image, I(p, q) is the potency of pixels (p, q) in genuine image and K(p, q) is the potency of pixels (p, q) in denoised image.

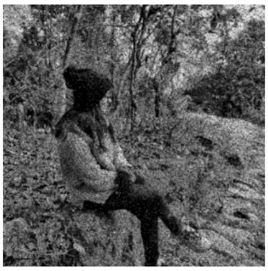
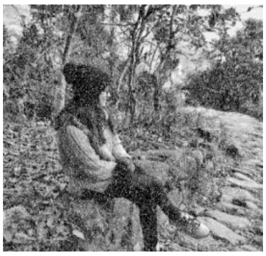
8. EXPERIMENTAL RESULTS

Wavelet and Shearlet transform by hard thresholding to demonstrate its potency. Experimental results were conducted to denoise a normal image as shown in fig. 8.1. Gaussian, salt & pepper and speckle noise were considered. Wavelet and Shearlet transform with hard thresholding used and their different denoised image is shown in table 8.1.

Table-8.1 PSNR and MSE Values

PSNR Value		
Noises	**Denoising using wavelet Transform**	**Denoising using shearlet Transform**
GAUSSIAN NOISE	15.0717032486	33.49161705261
SALT-PEPPER NOISE	13.94875911273	35.752284511
SPECKLE NOISE	16.0879278	32.73376986
MSE Value		
Noises	**Denoising using Wavelet Transform**	**Denoising using shearlet Transform**
GAUSSIAN NOISE	0.032240132	0.0004475466
SALT-PEPPER NOISE	0.0402832117	0.0002659326
SPECKLE NOISE	0.0311049204	0.000532872

Fig.8.1 Original Image

	NOISED IMAGE	DENOISED IMAGE USING WAVELET TRANSFORM	DENOISED IMAGE USING SHEARLET TRANSFORM
SPECKLE NOISE			
GAUSSIAN NOISE			
SALT & PEPPER NOISE			

CONCLUSION

This paper presents the denoising of a natural image based on wavelet and shearlet trans- form with hard thresholding techniques which is used to eliminate noise from the image. The images are corrupted with Gaussian, salt pepper and speckle noise. The multiscale and multi-directional outlook of shearlet transform are methodical in take care of edges of an image in denoising procedures. Shearlet comes out as a methodical transform for edges analysis and detection. Quantitative performance parameters such as PSNR, MSE are used to evaluated the denoised image effect. And hence came to the conclusion that the Shearlet Transform with hard thresholding in pyshear lab (python) is an methodical technique for enhancing the overall quality of the image.

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